PLANAR KNOTTING MECHANISMS FOR TURKISH HAND WOVEN CARPET

ME 332 THEORY OF MACHINES

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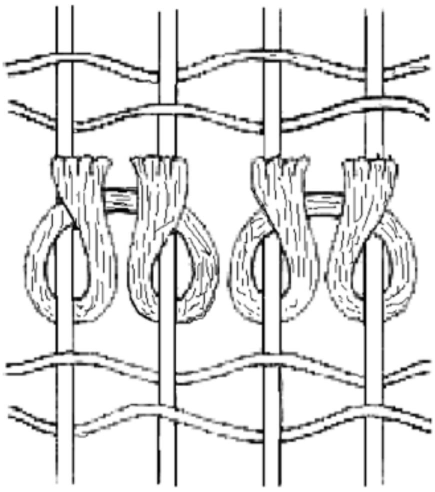
ABSTRACT

The problem is that too much time is needed when weaving a hand made carpet by hand. To solve this problem, a full automated electromechanical system is designed and developed. At first, parts of the system is considered and designed each them seperately. Secondly, kinematic,dynamic and force analysis calculations are made by the planar four-bar linkage of the system. Next step is manufacturing and choosing the material of the system. Delrin and the microservo motors are the main material of the base part and linkages,after that drilling and milling the delrin, microservo motors are put in the mechanism. Furthermore, gears of the mechanism are chosen and the ground part of the system is drilled according to those gears. Finally, mechanism is tested and solved the minor problems which may be ocur in the connecting parts such as bolts. As a result, using this kind of a full automated electromechanical system increase the product speed and there is no need the too much employer force to weave a conventional carpet.

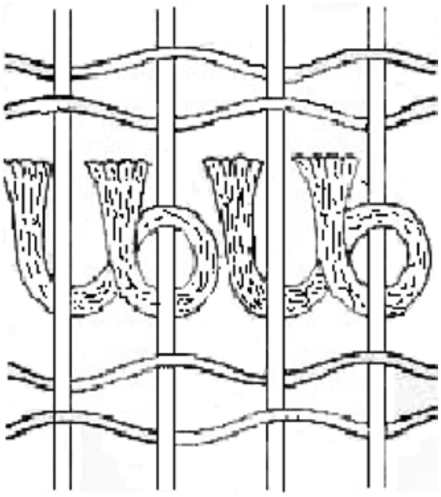
1-INTRODUCTION

Carpets can be classified according to their manufacturing methods. These methods are termed with respect to how they are produced. Some carpets are handmade, whereas the others are produced by automatic machinery. The number of carpet weaving looms which work at high speeds has increased along with improving technology over years. On the other hand, handmade carpets are woven by human hand due to the fact that the technology of handmade carpets has not changed over thousands of years. This is the problem that this paper will be interested in so that handmade carpets can be produced by a full automated electromechanical system. Hence, the traditional handmade carpets can be used more widely.

The texture of handmade carpets is formed from independent knots. In order to weave a handmade carpet, two types of knot are used; one is Turkish knot or double knot as shown in figure (a) and the other is Persian knot or single knot as can be seen from figure (b). The difference between these two knots is the Turkish knot yields a stronger and more durable carpet.



(a) The Turkish knot



(b) The Persian knot

This project aims to design and manufacture a planar knotting mechanism. There are thee separated parts to design the mechanism. First part is structural synthesis of mechanisms which consists of describing motion of working organ that is named as gripper, structural synthesis of mechanisms and animation of the technology process. Second part is to design linkage mechanisms. This part includes analytical synthesis of mechanisms and kinematical analysis of mechanisms; for instance, definition of position, linear velocity and acceleration on the working points of the link and definition of angular velocity and acceleration of the links. In third part, strength analysis of linkages will be figured out. Selection of material, calculation of the cross sectional area of links, kinematical analysis of mechanism with real mass of links and calculating real actuator force or moment, selection of the motor, selection of the other mechanical elements such as bearings, bolts and etc. will be taken into account in this part.

Finally, when everything is considered, we will manufacture the links according to technical drawing which is our design of the planar knotting mechanism.

2.INFORMATION OF CALCULATION PART:

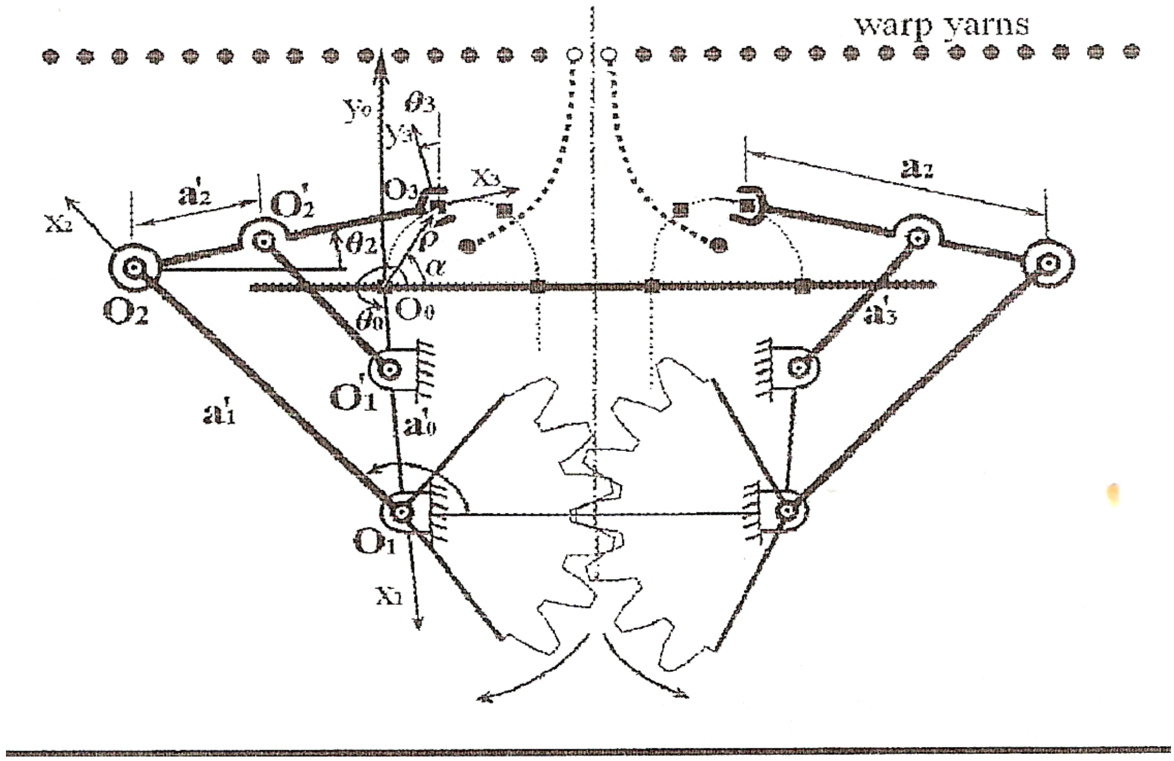


FIGURE -1 KNOTTING MECHANISMS WITH GEARS

Design Parameters:

1- a_0
2- θ_0
3- a_1
4- $\mathbf{a}_2$

Joint Parameters:

1- θ_1
2- θ_2
$\theta_3 = \theta_2$

Origin Coordinate System:

$O_1X_1Y_1$ and $O_2X_2Y_2$

Z_i is joint axes ==> Parallel to each other . So $\alpha_i = 0$ and $d_i = 0$. (α_i is twist angle, d_i is joint distance.)

θ_0 parameter assignment is arbitrary. Use superposition method finite solutions are available.

$$\overline{O_1O_2} = \overline{O_0O_3} - \overline{O_0O_1} - \overline{O_2O_3}$$

*complex polar notation

$$a_1 e^{j\theta_1} = \rho \ e^{j\alpha} - a_0 e^{j\theta_0} - a_2 e^{j\theta_2}$$

*writing differently

$$a_1(\cos\theta_1 + j\sin\theta_1) = \rho(\cos\alpha + j\sin\alpha) - a_0(\cos\theta_0 + j\sin\theta_0) - a_2(\cos\theta_2 + j\sin\theta_2)$$

*seperate real and imaginary parts

$$a_1\cos\theta_1 = \rho\cos\alpha - a_0\cos\theta_0 - a_2\cos\theta_2 \quad (\rho\cos\alpha \text{ is } \rho_x.)$$

$$a_1\sin\theta_1 = \rho\sin\alpha - a_0\sin\theta_0 - a_2\sin\theta_2 \quad (\rho\sin\alpha \text{ is } \rho_y.)$$

*take both sides square and divided together

$$a_1^2 - a_2^2 - a_0^2 + 2a_0(\rho_x\cos\theta_0 + \rho_y\sin\theta_0) + 2a_2(\rho_x\cos\theta_2 + \rho_y\sin\theta_2) - \rho^2 - 2a_0a_2\cos(\theta_2 - \theta_3)$$

$$(a_1^2 - a_2^2 - a_0^2)(a_0a_2)^{-1} + 2a_2^{-1}(\rho_{x_i}\cos\theta_0 + \rho_{y_i}\sin\theta_0) + 2a_0^{-1}(\rho_{x_i}\cos\theta_2 + \rho_{y_i}\sin\theta_2) + (a_0a_2)^{-1}(-\rho_i^2) - 2\cos(\theta_{2i} - \theta_0)$$

*i=1,2,3 set of positions.

*Three unknown==> a_0, a_1, a_2

$$P_1 f_{1_i} + P_2 f_{2_i} + P_3 f_{3_i} + P_4 f_{4_i} - F_i = 0 \qquad i = 1,2,3$$

$$P_1 = (a_1^2 - a_2^2 - a_0^2)(a_0 a_2)^{-1}$$

$$P_2 = a_2^{-1}$$

$$P_3 = a_0^{-1}$$

$$P_4 = (a_0 a_2)^{-1}$$

$$f_{1_i} = 1$$
$$f_{2_i} = 2(\rho_{xi} \cos\theta_0 + \rho_{yi} \sin\theta_0)$$
$$f_{3i} = 2(\rho_{xi} \cos\theta_{2i} + \rho_{yi} \sin\theta_{2i})$$
$$f_{4i} = -\rho_i^2 = -(\rho_{xi}^2 + \rho_{yi}^2)$$
$$F_i = 2(\cos\theta_{2i} - \theta_0)$$

$P_4 = P_2 P_3 = \lambda$ [Introduced eq. for upper nonlinear eq.] with this " P_4 " equation, we have 4 unknown and 4 equation

$P_1 f_{1i} + P_2 f_{2i} + P_3 f_{3i} = F_i - \lambda f_{4i} \quad i = 1,2,3$ (General Equation)

$P_2 P_3 - \lambda = 0$

In this equation constant parameters P_i are linear means that sythesis parameters are linearly proportional with non-linear parameters λ as follows

$P_k = l_k + \lambda M_k \qquad k = 1,2,3$ (l_k and M_k are reel nonlinear part)

Use this equation in general equation

$l_1 f_{1i} + l_2 f_{2i} + l_3 f_{3i} = F_i \qquad i = 1,2,3$ (use cramer's rule)

$M_1 f_{1i} + M_2 f_{2i} + M_3 f_{3i} = -f_{4i}$

Subst. $P_k = k + \lambda M_k$ for $k = 2,3$ ➔ give a second order equation

$$p\lambda^2 + q\lambda + r = 0$$

$$p = M_2 M_3$$

$$q = l_2 M_3 + l_3 M_2 - 1$$

$$r = l_2 l_3$$

In polinomial equation use P_k values :

$$a_0 = P_3^{-1}$$

$$a_1 = (P_1 P_2^{-1} P_3^{-1} + P_2^{-2} + P_3^{-2})^{0.5}$$

$$a_2 = P_2^{-1}$$

To solve the questions, there may be assumptions that are three precision:

1- ρ_x

2- ρ_y

3- θ_2

4- θ_0

Then calculate [f_{ki}], $k, i = 1,2,3$ column vector [F][f_4].

Up to know, i can tell how the calculations for design and manufacturing a planar knotting mechanism for Turkish hand woven carpet technology process is done. Now, we consider to solve an example about this issue.

CALCULATION PART:

Firstly, we can precise the four parameters ρ_x, ρ_y, θ_2, and θ_0 .

	ρ_x	ρ_y	θ_2	θ_0
s.1	0	0	10°	275°
s.2	25	15	3°	275°
s.3	30	1	-25°	275°

$$\begin{bmatrix} \rho_{1i} = 0 \\ \rho_{i2} = 29{,}154 \\ \rho_{i3} = 30{,}016 \end{bmatrix}$$

We can make up this table from interval that ➔ our values : [5 cm and 3 cm]

ρ_x ➔ $0 \le \rho_x \le 50mm$ $\qquad$ θ_2 ➔ $0 \le \theta_2 \le 90^\circ$

ρ_y ➔ $0 \le \rho_y \le 30mm$ $\qquad$ θ_0 ➔ $180^\circ \le \theta_0 \le 360^\circ$

General form of the equation which we use at the calculations:

$$\frac{a_1^2 - a_0^2 - a_2^2}{a_0 a_2} + \frac{2}{a_2}[\rho_{xi}\cos\theta_0 + \rho_{yi}\sin\theta_0] + \frac{2}{a_0}[\rho_{xi}\cos\theta_2 + \rho_{yi}\sin\theta_2] + \frac{[-\rho_i]^2}{a_0 a_2} - 2\cos[\theta_{2i} - \theta_0] = 0$$

$i = 1,2,3$

Situation-1

$$\frac{a_1^2 - a_0^2 - a_2^2}{a_0 a_2} + \frac{2}{a_2}[0.\cos 275^\circ + 0.\sin 275^\circ] + \frac{2}{a_0}[0.\cos 10^\circ + 0.\sin 10^\circ] + \frac{[0]^2}{a_0 a_2} - 2\cos(10 - 275)^\circ = 0$$

$a_1^2 - a_2^2 - a_0^2 + 0{,}174 a_0 a_2 = 0$ [1]

Situation-2

$$\frac{a_1^2 - a_0^2 - a_2^2}{a_0 a_2} + \frac{2}{a_2}[25.\cos 275^\circ + 15.\sin 275^\circ] + \frac{2}{a_0}[25.\cos 3^\circ + 15.\sin 3^\circ] + \frac{(-29{,}154)^2}{a_0 a_2} - 2\cos(-272^\circ) = 0$$

$a_1^2 - a_0^2 - a_2^2 - 25{,}528 a_0 + 51{,}49 a_2 + 849{,}955 - 0{,}069 a_0 a_2 = 0$ [2]

Situation-3

$$\frac{a_1^2 - a_0^2 - a_2^2}{a_0 a_2} + \frac{2}{a_2}[30.\cos 275^\circ + 1.\sin 275^\circ] + \frac{2}{a_0}[10.\cos(-25^\circ) + 1.\sin(-25^\circ)] + \frac{(-30{,}016)^2}{a_0 a_2} - 2\cos(-25 - 275) = 0$$

$a_1^2 - a_0^2 - a_2^2 + (3{,}236) a_0 + (53{,}534) a_2 + 900{,}96 - a_0 a_2 = 0$ [3]

1- $a_1^2 - a_2^2 - a_0^2 + 0{,}174 a_0 a_2 = 0$
2- $a_1^2 - a_2^2 - a_0^2 - 25{,}528 a_0 + 51{,}499 a_2 + 849{,}955 - 0{,}069 a_2 a_0 = 0$
3- $a_1^2 - a_2^2 - a_0^2 + 3{,}236 a_0 + 53{,}534 a_2 + 900{,}96 - a_a a_2 = 0$

Multiple by $\frac{1}{a_0 a_2}$;

1- $\frac{a_1^2 - a_2^2 - a_0^2}{a_0 a_2} + 0{,}174 = 0$

2- $\frac{a_1^2 - a_2^2 - a_0^2}{a_0 a_2} - \frac{25{,}528}{a_2} + \frac{51{,}499}{a_0} + \frac{849{,}955}{a_0 a_2} - 0{,}069 = 0$

3- $\frac{a_1^2 - a_2^2 - a_0^2}{a_0 a_2} + \frac{3{,}236}{a_2} + \frac{53{,}534}{a_0} + \frac{900{,}96}{a_0 a_2} - 1 = 0$

$$0+0+P_1+0=0{,}174$$

$$-25{,}528P_2+51{,}499P_3+P_1+849{,}955P_4=0{,}069$$

$$3{,}236P_2+53{,}534P_3+P_1+900{,}96P_4=1$$

Write it

$$(l_2+\lambda M_2)(0)+(l_3+\lambda M_3)(0)+(l_1+\lambda M_1)+0=-0{,}174$$

$$(l_2+\lambda M_2)(-25{,}528)+(l_3+\lambda M_3)(51{,}499)+(l_1+\lambda M_1)+849{,}955\lambda=0.069$$

$$(l_2+\lambda M_2)(3{,}236)+(l_3+\lambda M_3)(53{,}534)+(l_1+\lambda M_1)+900{,}96\lambda=1$$

Write the l part and M part differently

$$l_1+0+0=-0{,}174$$

$$l_1-25{,}528l_2+51{,}499l_3=0{,}069$$

$$l_1+3{,}236l_2+53{,}534l_3=1$$

$$\begin{bmatrix}1 & 0 & 0\\ 1 & -25{,}528 & 51{,}499\\ 1 & 3.236 & 53{,}534\end{bmatrix}\begin{bmatrix}l_1\\ l_2\\ l_3\end{bmatrix}=\begin{bmatrix}0{,}174\\ 0{,}069\\ 1\end{bmatrix}$$

$$M_1+0+0=0$$

$$M_1-25{,}528M_2+51{,}499M_3=-849{,}955$$

$$M_1+3.236M_2+53{,}534M_3=-900{,}96$$

$$\begin{bmatrix}1 & 0 & 0\\ 1 & -25{,}528 & 51{,}499\\ 1 & 3.236 & 53{,}534\end{bmatrix}\begin{bmatrix}M_1\\ M_2\\ M_3\end{bmatrix}=\begin{bmatrix}0\\ -849{,}955\\ -900{,}96\end{bmatrix}$$

From the matrix equation we can find;

$l_1=-0{,}174$ $\qquad$ $m_1=0$

$l_2=0{,}0309$ $\qquad$ $m_2=-0{,}585$

$l_3=0{,}02$ $\qquad$ $m_3=-16{,}79$

$p = m_2 m_3 = (-0{,}585)(-16{,}79) = 9{,}822$

$q = l_2 m_3 + l_3 m_2 - 1 = (0{,}0309)(-16{,}79) + (0{,}02)(-0{,}585) - 1 = -1{,}5305$

$r = l_2 l_3 = (0{,}0309)(0{,}02) = 6{,}18 * 10^{-4}$

$p\lambda^2 + q\lambda + r = 0$

$9{,}822\lambda^2 + (-1{,}5305)\lambda + 0{,}000618 = 0$

$\Delta = b^2 - 4ac \Rightarrow \Delta = (-1{,}5305)^2 - 4.(9{,}822).(0{,}000618) = 2{,}317 > 0$

$\Delta > 0$ so there are two real roots.

$$\lambda_{1,2} = \frac{-b \mp \sqrt{\Delta}}{2a} \quad \Rightarrow \quad \lambda_{1,2} = \frac{-(-1{,}5305) \mp \sqrt{2{,}317}}{2.(9{,}822)}$$

$\lambda_1 = 0{,}155$ and $\lambda_2 = 0{,}000432$

For $\lambda_1 = 0{,}155$:

$p_1 = l_1 + \lambda m_1 = (-0{,}174) + (0{,}155).0 = -0{,}174$

$p_2 = l_2 + \lambda m_2 = (0{,}0309) + (0{,}155).(-0{,}585) = -0{,}059$

$p_3 = l_3 + \lambda m_3 = 0{,}02 + (0{,}155)(-16{,}79) = -2{,}582$

**CASE 1

$$a_0 = p_3^{-1} = \frac{1}{p_3} = \frac{1}{-2{,}582} = -0{,}387$$

$$a_2 = p_2^{-1} = \frac{1}{p_2} = \frac{1}{-0{,}059} = -16{,}949$$

$$a_1 = [p_1 p_2^{-1} p_3^{-1} + p_2^{-2} + p_3^{-2}] \Rightarrow a_1 = [-\frac{0{,}274}{0{,}152} + \frac{1}{0{,}003481} + \frac{1}{6{,}666}]^{0,5} = 16{,}919$$

For $\lambda_2 = 0{,}000432$

$$p_1 = l_1 + \lambda_2 m_1 = (-0{,}174) + (0{,}000432).0 = -0{,}174$$

$$p_2 = l_2 + \lambda_2 m_2 = 0{,}0309 + (0{,}000432)(-0{,}585) = 0{,}03064$$

$$p_3 = l_3 + \lambda_2 m_3 = 0{,}02 + (0{,}000432)(-16{,}79) = 0{,}01274$$

**CASE2

$$a_0 = p_3^{-1} = \frac{1}{0{,}01274} = 78{,}492$$

$$a_2 = p_2^{-1} = \frac{1}{0{,}02064} = 32{,}637$$

$$a_1 = [\frac{p_1}{p_2 p_3} + \frac{1}{p_2^2} + \frac{1}{p_3^2}]^{0,5} \Rightarrow a_1 = [-\frac{0{,}174}{(0{,}03064)(0{,}01274)} + \frac{1}{(0{,}03064)^2} + \frac{1}{(0{,}01274)^2}]^{1/2} = 82{,}345$$

**RESULTS

$$a_0' = (a_0)_1 - (a_0)_2 = 78{,}879$$

$$a_1' = (a_1)_1 - (a_1)_2 = 65{,}426$$

$$a_2' = (a_2)_1 - (a_2)_2 = 15{,}688$$

3-KINEMATIC ANALYSIS OF MECHANISM:

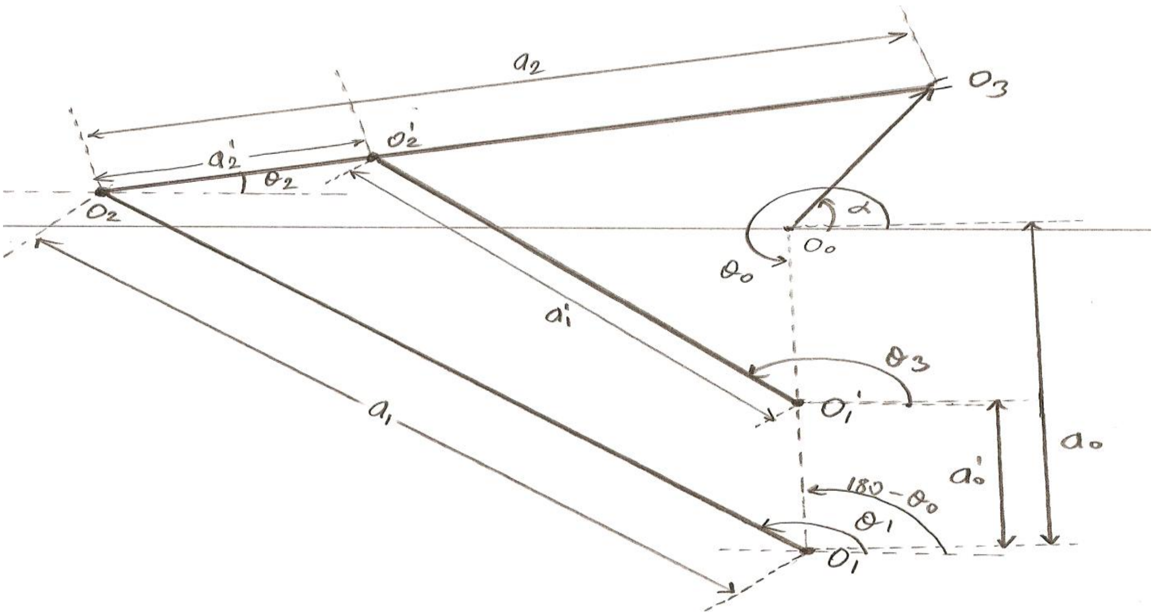
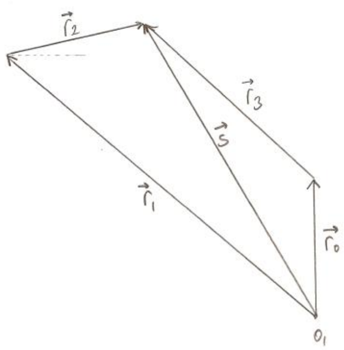


FIGURE -2 ONE SIDE OF MECHANISM

Parameters :

$\lvert O_1O_1'\rvert = a_0' = r_0$	,	$\vec{r}_0 = a_0'\angle(180-\theta_0)$
$\lvert O_1O_2\rvert = a_1 = r_1$	,	$\vec{r}_1 = a_1\angle\theta_1$
$\lvert O_2O_2'\rvert = a_2' = r_2$	,	$\vec{r}_2 = a_2'\angle\theta_2$
$\lvert O_2'O_1'\rvert = a_1' = r_3$	,	$\vec{r}_3 = a_1'\angle\theta_3$

Draw a line between O_1 and O_2' ➔ $\lvert O_1O_2'\rvert = s$, $\vec{s} = s\angle\theta_s$



Loop Closure Equations

$$\vec{r}_1 + \vec{r}_2 = \vec{s}$$

$$\vec{s} = \vec{r}_0 + \vec{r}_3$$

FIGURE -3 VECTORAL SHOWN OF MECHANISM

$$r_1 e^{j\theta_1} + r_2 e^{j\theta_2} = s e^{j\theta_s}$$

$r_1 \cos\theta_1 + r_2 \cos\theta_2 = s\cos\theta_s$ ➔ real part

$r_1 \sin\theta_1 + r_2 \sin\theta_2 = s\sin\theta_s$ ➔ imaginary part

By taking squares of to equations and adding them each other, we will get;

$$r_1^2 + 2r_1 r_2 c\theta_1 c\theta_2 + r_2^2 + 2r_1 r_2 s\theta_1 s\theta_2 = s^2$$

$$r_1^2 + r_2^2 + 2r_1 r_2 (c\theta_1 c\theta_2 + s\theta_1 s\theta_2) = s^2$$

$$r_1^2 + r_2^2 + 2r_1 r_2 \cos(\theta_1 - \theta_2) = s^2$$

➔ $s = \sqrt{r_1^2 + r_2^2 + 2r_1 r_2 \cos(\theta_1 - \theta_2)}$

$$\tan\theta_s = \frac{r_1 \sin\theta_1 + r_2 \sin\theta_2}{r_1 \cos\theta_1 + r_2 \cos\theta_2}$$

➔ $\theta_s = \tan^{-1}[\frac{(r_1 \sin\theta_1 + r_2 \sin\theta_2)}{(r_1 \cos\theta_1 + r_2 \cos\theta_2)}]$

➔ $\theta_1 = \tan^{-1}[\frac{s\sin\theta_s - r_2 \sin\theta_2}{s_2 \cos\theta_s - r_2 \cos\theta_2}]$

$$s e^{j\theta_s} = r_0 e^{j(180-\theta_0)} + r_3 e^{j\theta_3}$$

➔ $s = r_0 e^{j(180-\theta_0-\theta_s)} + r_3 e^{j(\theta_3-\theta_s)}$

➔ $s = r_0 \cos(180 - \theta_0 - \theta_s) + r_3 \cos(\theta_3 - \theta_s)$

➔ $0 = r_0 \sin(180 - \theta_0 - \theta_s) + r_3 \sin(\theta_3 - \theta_s)$

$$\cos(180 - (\theta_0 + \theta_s)) = -\cos(\theta_0 + \theta_s)$$

$$\sin(180 - (\theta_0 + \theta_s)) = \sin(\theta_0 + \theta_s)$$

1- $r_3 \cos(\theta_3 - \theta_s) = s + r_0 \cos(\theta_0 + \theta_s)$

$r_3 \sin(\theta_3 - \theta_5) = -r_0 \sin(\theta_0 + \theta_s)$

➔ $r_3^2 = s^2 + 2sr_0 \cos(\theta_0 + \theta_s) + r_0^2$

➔ $\cos(\theta_0 + \theta_s) = \frac{r_3^2 - s^2 - r_0^2}{2sr_0}$

➔ $\theta_s = -\theta_0 \pm \cos^{-1}[\frac{r_3^2 - s^2 - r_0^2}{2sr_0}]$

2- $s - r_3 \cos(\theta_3 - \theta_s) = -r_0 \cos(\theta_0 + \theta_s)$

$r_3 \cos(\theta_3 - \theta_s) = -r_0 \sin(\theta_0 + \theta_s)$

➔ $r_0^2 = s^2 - 2sr_3 \cos(\theta_3 - \theta_s) + r_3^2$

➔ $\cos(\theta_3 - \theta_s) = \frac{s^2 + r_3^2 - r_0^2}{2sr_3}$

➔ $\theta_3 = \theta_s \pm \cos^{-1}[\frac{s^2 + r_3^2 - r_0^2}{2sr_3}]$

After obtaining all unknowns:

Loop closure equation; $\vec{r}_1 + \vec{r}_2 = \vec{r}_0 + \vec{r}_3$

➔ $r_1 e^{j\theta_1} + r_2 e^{j\theta_2} = r_0 e^{j(180-\theta_0)} + r_3 e^{j\theta_3}$

➔ $r_1 e^{j\theta_1} + r_1 \theta_1 j e^{j\theta_1} + r_2 e^{j\theta_2} + r_2 \theta_2 j e^{j\theta_2} = r_0 e^{j(180-\theta_0)} + r_0 \theta_3 j e^{j(180-\theta_0)} + r_3 e^{j\theta_3} + r_3 \theta_3 j e^{j\theta_3}$

➔ $r_1 \omega_1 e^{j\theta_1} + r_2 \omega_2 e^{j\theta_2} = r_3 \omega_3 e^{j\theta_3}$

ω_2 and ω_3 are unknowns.

$r_1 \omega_1 e^{j(\theta_1-\theta_2)} + r_2 \omega_2 = r_3 \omega_3 e^{j(\theta_3-\theta_2)}$

$r_1 \omega_1 \cos(\theta_1 - \theta_2) + r_2 \omega_2 = r_3 \omega_3 \cos(\theta_3 - \theta_2)$ ➔ real part

$r_1 \omega_1 \sin(\theta_1 - \theta_2) = r_3 \omega_3 \sin(\theta_3 - \theta_2)$ ➔ imaginary part

➔ $\omega_3 = (\frac{r_1 \omega_1}{r_3}) \frac{\sin(\theta_1 - \theta_2)}{\sin(\theta_3 - \theta_2)}$

$r_1 \omega_1 \cos(\theta_1 - \theta_3) + r_2 \omega_2 \cos(\theta_2 - \theta_3) = r_3 \omega_3$ ➔ real part

$r_1 \omega_1 \sin(\theta_1 - \theta_3) + r_2 \omega_2 \sin(\theta_2 - \theta_3) = 0$ ➔ imaginary part

➔ $\omega_2 = -(\frac{r_1 \omega_1}{r_2}) \frac{\sin(\theta_1 - \theta_3)}{\sin(\theta_2 - \theta_3)}$

Loop closure equation; $r_1 \omega_1 e^{j\theta_1} + r_2 \omega_2 e^{j\theta_2} = r_3 \omega_3 e^{j\theta_3}$

$r_1 \alpha_1 e^{j\theta_1} + (r_1 \omega_1) j \omega_1 e^{j\theta_1} + r_2 \alpha_2 e^{j\theta_2} + (r_2 \omega_2) j \omega_2 e^{j\theta_2} = r_3 \alpha_3 e^{j\theta_3} + (r_3 \omega_3) j \omega_3 e^{j\theta_3}$

$e^{j(\theta_1-\theta_3)} (r_1 \alpha_1 + r_1 \omega_1^2 j) + e^{j\theta_2} (r_2 \alpha_2 + r_2 \omega_2^2 j) = e^{j\theta_3} (r_3 \alpha_3 + r_3 \omega_3^2 j)$

1- $e^{j(\theta_1-\theta_3)} (r_1 \alpha_1 + r_1 \omega_1^2 j) + e^{j(\theta_2-\theta_3)} (r_2 \alpha_2 + r_2 \omega_2^2 j) = (r_3 \alpha_3 + r_3 \omega_3^2 j)$

İmaginary part ;

$$r_1\omega_1^2 c(\theta_1-\theta_3)+r_1\alpha_1 s(\theta_1-\theta_3)+r_2\omega_2^2 c(\theta_2-\theta_3)+r_2\alpha_2 s(\theta_2-\theta_3)=r_3\omega_3^2$$

$$\Rightarrow \alpha_2=\frac{r_3\omega_3^2-r_1\omega_1^2\cos(\theta_1-\theta_3)-r_2\omega_2\cos(\theta_2-\theta_3)}{r_2\sin(\theta_2-\theta_3)}$$

2- $e^{j(\theta_1-\theta_2)}(r_1\alpha_1+r_1\omega_1^2 j)+(r_2\alpha_2+r_2\omega_2^2 j)=e^{j(\theta_3-\theta_2)}(r_3\alpha_3+r_3\omega_3^2 j)$

İmaginary part;

$$r_1\omega_1^2 c(\theta_1-\theta_2)+r_1\alpha_1 s(\theta_1-\theta_2)+r_2\omega_2^2=r_3\omega_3^2 c(\theta_3-\theta_2)+r_3\alpha_3 s(\theta_3-\theta_2)$$

$$\alpha_3=\frac{r_1\omega_1^2\cos(\theta_1-\theta_2)+r_2\omega_2^2-r_3\omega_3^2\cos(\theta_3-\theta_2)}{r_3\sin(\theta_3-\theta_2)}$$

4-DYNAMIC ANALYSIS

All of required equations have been found in kinematic analysis part. There are 7 required parameters. These are $s,\theta_s,\theta_3,\omega_3,\omega_2,a_2,a_3$. To find these unknowns, link lenghts and some angles are found or decided by assuming before. These are;

$$\vec{r}_0=a_0'\angle(180-\theta_0),\theta_0=275^\circ,a_0'=78{,}105mm$$

$$\vec{r}_1=a_1\angle\theta_1,\theta_1=135^\circ,a_1=82{,}345mm$$

$$\vec{r}_2=a_2'\angle\theta_2,\theta_2=10^0,a_2'=15{,}688mm$$

$$\vec{r}_3=a_1'\angle\theta_3,\theta_3=?,a_1'=65{,}426mm$$

$$s=[r_1^2+r_2^2+2r_1r_2\cos(\theta_1-\theta_2)]^{1/2} \Longrightarrow s=[(82{,}345)^2+(15{,}688)^2+2(82{,}345)(15{,}688)\cos(135-10)^\circ]=74{,}464mm$$

$$\theta_s=\tan^{-1}[\frac{r_1\sin\theta_1+r_2\sin\theta_2}{r_1\cos\theta_1+r_2\cos\theta_2}] \Longrightarrow \theta_s=\tan^{-1}[\frac{(82{,}345)\sin 135^\circ+(15{,}688)\sin 10^\circ}{(82{,}345)\cos 135^\circ+(15{,}688)\cos 10^\circ}]=\tan^{-1}[\frac{60{,}95}{-42{,}777}]=125^\circ$$

$$\theta_3 = \theta_s \pm \cos^{-1}[\frac{s^2 + r_3^2 - r_0^2}{2sr_3}] ==> \theta_3 = 125^\circ \pm \cos^{-1}[\frac{(74{,}464)^2 + (65{,}426)^2 - (78{,}105)^2}{2(74{,}464)(65{,}426)}] = 125^\circ \pm \cos^{-1}[\frac{3725{,}058}{9743{,}763}] = 192{,}524$$

$$\theta_3 = (\frac{r_1\omega_1}{r_3})\frac{\sin(\theta_1 - \theta_2)}{\sin(\theta_3 - \theta_2)} ==> \omega_3 = \frac{(82{,}345)\omega_1}{65{,}426}\frac{\sin(135-10)^\circ}{\sin(192{,}524-10)^\circ} ==> \omega_3 = -23{,}41\omega_1 \text{ cw}$$

ω_1 is in counter clockwise (+) direction, ω_3 is in clockwise (-) direction.

$$\omega_2 = -(\frac{r_1\omega_1}{r_2})\frac{\sin(\theta_1 - \theta_3)}{\sin(\theta_2 - \theta_3)} ==> \omega_2 = -\frac{(82{,}345)\omega_1}{15{,}688}\frac{\sin(135-192{,}524)^\circ}{\sin(10-192{,}524)^\circ} ==> \omega_2 = 100{,}55\omega_1 \text{ ccw}$$

$$\alpha_2 = \frac{r_3\omega_3^2 - r_1\omega_1^2\cos(\theta_1-\theta_3) - r_2\omega_2^2\cos(\theta_2-\theta_3)}{r_3\sin(\theta_3-\theta_2)} = \frac{35855{,}29\omega_1^2 - 44{,}215\omega_1^2 - (-158456{,}55\omega_1^2)}{-2{,}88} = -65454{,}04\omega_1^2$$

$$\alpha_3 = \frac{r_1\omega_1^2\cos(\theta_1-\theta_2) + r_2\omega_2^2 - r_3\omega_3^2\cos(\theta_3-\theta_2)}{r_3\sin(\theta_3-\theta_2)}$$

$$\alpha_3 = \frac{(82{,}345)\omega_1^2\cos(135-10 + (15{,}688)(100{,}55\omega_1)^2 - (65{,}426)(-23{,}41\omega_1)^2\cos(192{,}524-10)^\circ}{(65{,}426\sin(192{,}524-10)^\circ}$$

$$\alpha_3 = \frac{-47{,}23\omega_1^2 + 158610{,}43\omega_1^2 + 35820{,}5\omega_1^2}{-2{,}88} = -67494{,}34\omega_1^2$$

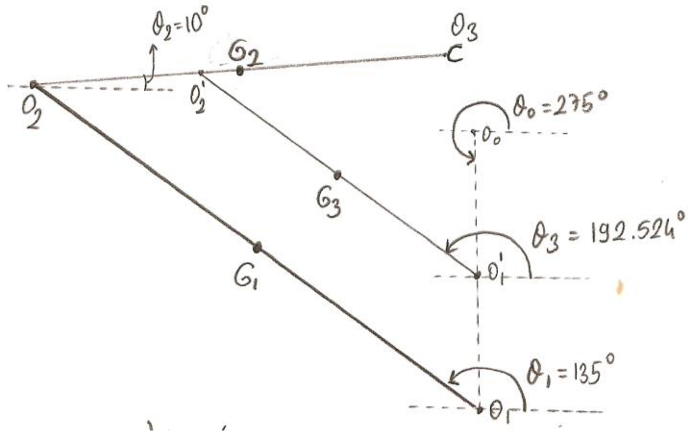


FIGURE -4 GRIPPER AND TWO LINKAGES

$$R_{o_1o_2} = (82{,}345\cos 135^\circ)i + (82{,}345\sin 135^\circ)j = -58{,}23i + 58{,}23j$$

$$R_{o_1G_1} = -29{,}115i + 29{,}115j$$

$$R_{o_2o_3} = (32{,}637\cos 10^\circ)i + (32{,}637\sin 10^\circ)j = 32{,}14i + 5{,}67j$$

$$R_{o_2G_2} = 16{,}07i + 2{,}83j$$

$$R_{o_1'o_2'} = (65{,}426\cos(192{,}524))i + (65{,}426\sin(192{,}524))j = -63{,}87i - 14{,}19j$$

$$R_{o_1'G_3} = -31{,}935i - 7{,}095j$$

CALCULATION OF ACCELERATIONS;

$\vec{a}_{G_1} = \vec{a}_{o_1} + \vec{\alpha}_1 \times \vec{R}_{o_1G_1} + \vec{\omega}_1 \times (\omega_1 \times \vec{R}_{o_1G_1})$ ➔ crank rotates at constant speed

$$\vec{a}_{G_1} = (\omega_1\hat{k}) \times ((\omega_1\hat{k}) \times (-29{,}115i + 29{,}115j)) ==> \vec{a}_{G_1} = (\omega_1\hat{k}) \times (-29{,}115\omega_1 j - 29{,}115\omega_1 i) = 29{,}115\omega_1^2 i - 29{,}115\omega_1^2 j$$

$$\vec{a}_{o_2} = \vec{a}_{o_1} + \vec{\alpha}_1 \times \vec{R}_{o_1o_2} + \vec{\omega}_1 \times (\vec{\omega}_1 \times \vec{R}_{o_1o_2}) ==> \vec{a}_{o_2} = (\omega_1\hat{k}) \times ((\omega_1\hat{k}) \times (-58{,}23i + 58{,}23j)) = 58{,}23\omega_1^2 i - 58{,}23\omega_1^2 j$$

$$\vec{a}_{G_2} = \vec{a}_{o_2} + \alpha_2 \times \vec{R}_{o_2G_2} + \omega_2 \times (\omega_2 \times \vec{R}_{o_2G_2})$$

$$\vec{a}_{G_2} = (58{,}23\omega_1^2 i - 58{,}23\omega_1^2 j) + (-65454{,}04\omega_1 \hat{k}) \times (16{,}07i + 2{,}83j) + (100{,}55\omega_1 \hat{k}) \times ((100{,}55\omega_1 \hat{k}) \times (16{,}07i + 2{,}83j))$$

$$\vec{a}_{G_2} = (58{,}23\omega_1^2 i - 58{,}23\omega_1^2 j) + (-1051846{,}4\omega_1^2 j + 185235\,\omega_1^2 i) + (-162472{,}56\omega_1^2 i - 28612{,}16\omega_1^2 j)$$

$$\vec{a}_{G_2} = 22820{,}67\omega_1^2 i - 1080517\,\omega_1^2 j$$

$$\vec{a}_{G_3} = \vec{a}_{o_1'} + \vec{\alpha}_3 \times \vec{R}_{o_1'G_3} + \vec{\omega}_3 \times (\vec{\omega}_3 \times \vec{R}_{o_1'G_3})$$

$$\vec{a}_{G_3} = (-67494{,}34\omega_1^2 \hat{k})(-31{,}935i - 7{,}095j) + (-23{,}41\omega_1 \hat{k}) \times ((-23{,}41\omega_1 \hat{k}) \times (-31{,}935i - 7{,}095j))$$

$$\vec{a}_{G_3} = [(2155431{,}75j - 478872{,}34i) + (17501{,}28i + 3888{,}26j)]\omega_1^2$$

$$\vec{a}_{G_3} = -461371{,}06\omega_1^2 i + 2159320\,\omega_1^2 j$$

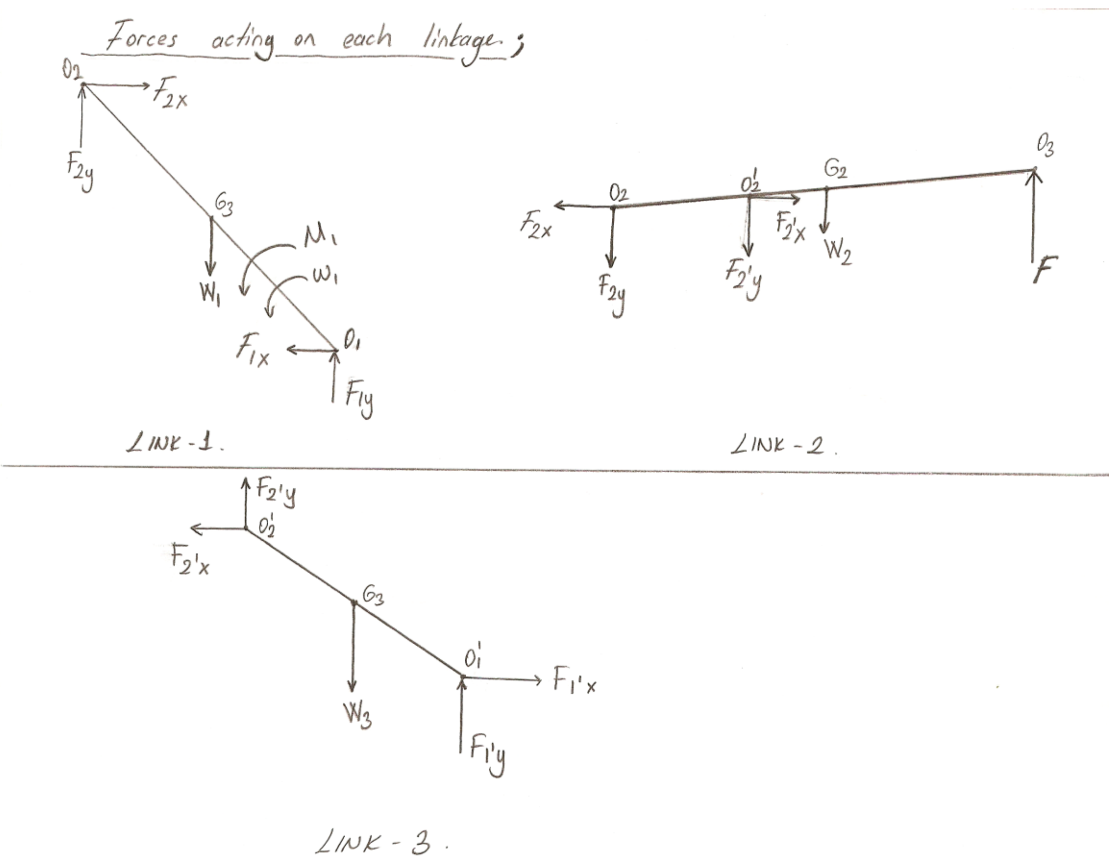


FIGURE-5 FORCES ACTING ON EACH LINK

5-FORCES ACTING ON LINKAGES

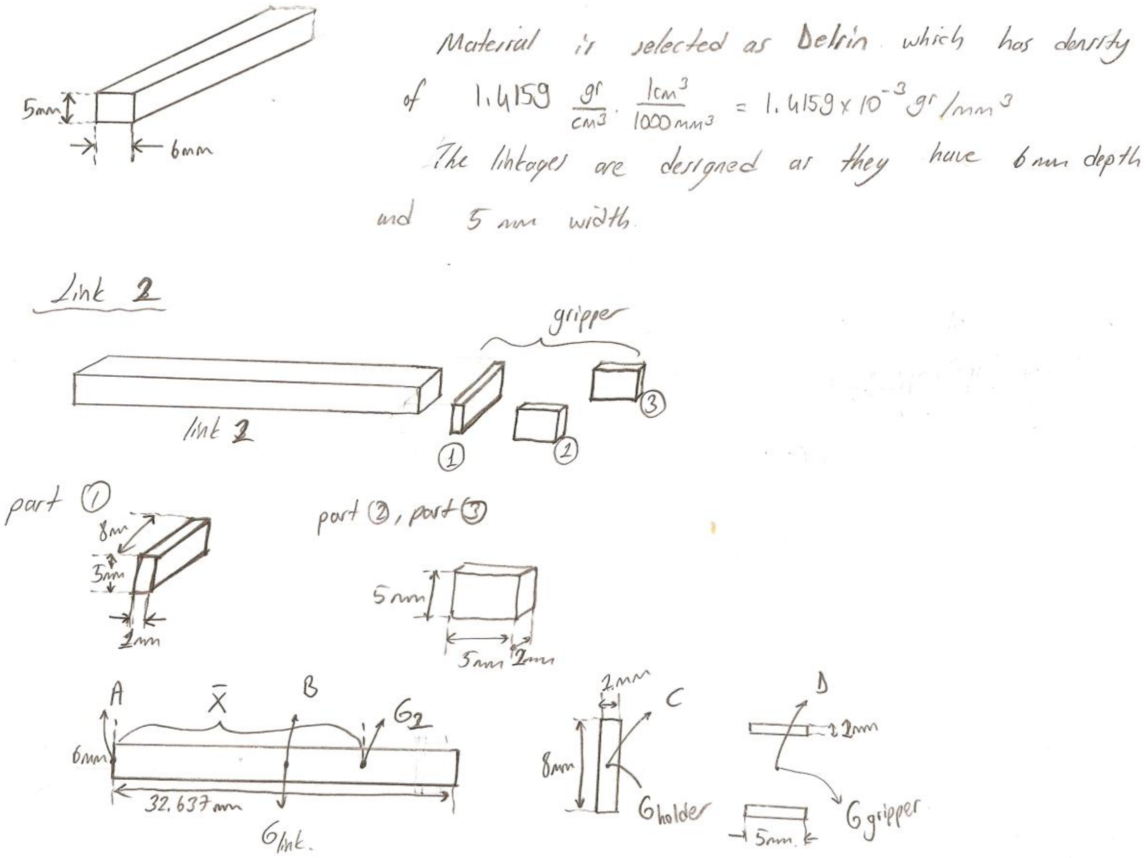


FIGURE-6 FORCES ACTING ON LINKS

$V_{link} = (6mm)(5mm)(32{,}637)$, $V_{holder} = (2mm)(8mm)(5mm)$,
$V_{gripper} = 2.(2mm)(5mm)(5mm)$

$V_{link} = 979{,}11mm^3$, $V_{holder} = 80mm^3$, $V_{gripper} = 100mm^3$

➔ $m_{link} = V_{link}\,.\rho = (979{,}11mm^3).(1{,}4159 \times 10^{-3}\ gr/mm^3) = 1{,}386\,gr$

➔ $m_{holder} = V_{holder}\,.\rho = (80mm^3)(1{,}4159 \times 10^{-3}\ gr/mm^3) = 0{,}113\,gr$

➔ $m_{gripper} = V_{gripper}\rho = (100mm^3)(1{,}4159 \times 10^{-3}\ gr/mm^3) = 0{,}142\,gr$

$$\bar{x} = \frac{m_{link}.|AB| + m_{holder}.|AC| + m_{gripper}.|AD|}{m_{link} + m_{holder} + m_{gripper}} = \frac{22{,}61 + 3{,}80 + 5{,}27}{1{,}641} = 19{,}30mm$$

For link 2 ➔ $m_2 = m_{link} + m_{holder} + m_{gripper} = 1{,}641 gr$

$$W_2 = m_2 .g = 1{,}641 \times 9{,}81 = 16{,}1 \times 10^{-2} N$$

$$I_{G_2} = I_{G_{link}} + m_{link} \left|BG_2\right|^2 + I_{G_{holder}} + m_{holder} \left|G_2 C\right|^2 + I_{G_{gripper}} + m_{gripper} \left|G_2 D\right|$$

$$I_{G_{link}} = \frac{m_{link}}{12}(6^2 + 32{,}637^2), I_{G_{holder}} = \frac{m_{holder}}{12}(8^2 + 2^2), I_{G_{gripper}} = \frac{m_{gripper}}{12}(5^2 + 4^2)$$

$$I_{G_2} = (127{,}1 \times 10^{-9} kg.m^2) + 12{,}32 \times 10^{-9} + 0{,}64 \times 10^{-9} + 23{,}22 \times 10^{-9} + 0{,}49 \times 10^{-9} + 45{,}17 \times 10^{-9} = 208{,}94 \times 10^{-9}$$

For link 1 ➔ $m_1 = V_1 .\rho$

$$V_1 = (6)(5)(82{,}345 mm) = 2470{,}35 mm^3 ==> m_1 = 3{,}5 gr ==> W_1 = m_1 .g = 34{,}31 \times 10^{-3} N$$

$$I_{G_1} = \frac{m_1}{12}(6^2 + 82{,}345^2) = 1988{,}20 \times 10^{-9} kgm^2$$

For link 3 ➔ $m_3 = V_3 .\rho ==> V_3 = (6)(5)(65{,}426) = 1962{,}78 mm^3$

$$m_3 = 2{,}78 gr ==> W_3 = m_3 g = 27{,}26 \times 10^{-3} N$$

$$I_{G_3} = \frac{m_3}{12}(6^2 + 65{,}426^2) = 1000 \times 10^{-9} kgm^2$$

<u>LINK 3</u>

$$\sum F = 0$$

$$\vec{F}_{2'} + \vec{W}_3 + \vec{F}_{1'} = m_3 \vec{a}_3$$

$$-F_{2'x} i + F_{2'y} j - W_3 j + F_{1'x} i + F_{1'y} j = m_3 \vec{a}_{G_3}$$

$$-F_{2'x} i + F_{2'y} j - (27{,}26 \times 10^{-3}) j + F_{1'x} i + F_{1'y} j = (2{,}78 \times 10^{-3} kg)(-461371{,}06 \omega_1^2 i + 2159320\, \omega_1^2 j)$$

$$-F_{2'x} i + F_{2'y} j - 27{,}26 \times 10^{-3} j + F_{1'x} + F_{1'y} j = -1282{,}6 \omega_1^2 i + 6002{,}9 \omega_1^2 j \quad \textbf{[1]}$$

$$\sum M_{o_i} = 0$$

$$\vec{R}_{G_3 O_i} \times (\vec{W}_3 + (-m_3 \vec{a}_{G_3})) + \vec{R}_{o'_2 o'_1} \times \vec{F}_{2'} + (-I_{G_3} \vec{\alpha}_3) = 0$$

$$(-31{,}9 i - 7{,}1 j) \times (-27{,}26 \times 10^{-3} j + 1282{,}6 \omega_1^2 i - 6002{,}9 \omega_1^2 j) + (-63{,}87 i - 14{,}19 j) \times (-F_{2'x} i + F_{2'y} j) + (I_{G_3} \alpha_3) = 0$$

$$(0{,}869 + 191{,}5 \times 10^3 \omega_1^2) \hat{k} + (9106{,}46 \omega_1^2) \hat{k} - 63{,}87 F_{2'y} \hat{k} - 14{,}19 F_{2'x} \hat{k} + 67494{,}34 \omega_1^2 \times 10^{-6} \hat{k} = 0$$

$$-14{,}19F_{2'x} - 63{,}87F_{2'y} + 0{,}869 + 2\times10^{5}\omega_1^2 = 0 \quad \textbf{[2]}$$

<u>LINK 2</u>

$$\sum\vec{F} = 0$$

$\vec{F} + \vec{F}_2 + \vec{W}_2 + \vec{F}_{2'} = m_2\vec{a}_{G_2}$, assume $\vec{F} = 15Nj$

$$15Nj - F_{2x}i - F_{2y}j + F_{2'x}i - F_{2'y}j - W_2 j = (1{,}641\times10^{-3})(22820{,}67\omega_1^2 i - 1080517\,\omega_1^2 j)$$

$$\sum\vec{M}_{o_2} = 0$$

$$\vec{R}_{o_3o_2}\times\vec{F} + \vec{R}_{G_2o_2}\times\vec{W}_2 + \vec{R}_{0'_2o_2}\times\vec{F}_{2'} + (-I_{G_2}\alpha_2) = 0$$

$$(32{,}14i + 5{,}67j)\times(15Nj) + (16{,}7i + 2{,}8j)\times(-16{,}1\times10^{-3}Nj) + (15{,}45i + 2{,}72j)\times(F_{2'x}i + F_{2'y}j)$$
$$+ (208{,}94\times10^{-9}\times65454{,}04\omega_1^2) = 0$$

$$(482{,}1\hat{k}) - 0{,}268\hat{k} + 15{,}45F_{2'y}\hat{k} - 2{,}72F_{2'x}\hat{k} + 0{,}014\omega_1^2 = 0 \quad \textbf{[3]}$$

Equation 2 ➔ $-14{,}19F_{2'x} - 63{,}87F_{2'y} = 0{,}869 + 2\times10^{5}\omega_1^2$

Equation 3 ➔ $2{,}72F_{2'x} - 15{,}45F_{2'y} = 481{,}83 + 0{,}014\omega_1^2$

Solving These equations.

$$F_{2'x} = \frac{\begin{vmatrix}0{,}869 + 2\times10^{5}\omega_1^2 & 63{,}87\\ 481{,}83 + 0{,}014\omega_1^2 & -15{,}45\end{vmatrix}}{\begin{vmatrix}14{,}19 & 63{,}87\\ 2{,}72 & -15{,}45\end{vmatrix}} \Rightarrow F_{2'x} = \frac{30787{,}9 + 3{,}1\times10^{6}\omega_1^2}{392{,}96}$$

Substitute $F_{2'x}$ into equation 2 to get $F_{2'y}$

$$F_{2'y} = -17{,}4 + 1378{,}7\omega_1^2$$

Substitute $F_{2'x}$ and $F_{2'y}$ into equation 1

$$-F_{2'x}i + F_{2'y}j - 27{,}26\times10^{-3}j + F_{1'x}i + F_{1'y}j = -1282{,}6\omega_1^2 i + 6002{,}9\omega_1^2 j$$

$$-F_{2'x}+F_{1'x}=-1282{,}6\omega_1^2$$

$$F_{1'x}=-78{,}34+6606{,}2\omega_1^2$$

$$F_{2'y}-27{,}26\times10^{-6}+F_{1'y}=6002{,}9\omega_1^2$$

$$F_{1'y}=17{,}4+4624{,}2\omega_1^2$$

For Link 2 $\sum\vec{F}=0$

$$15Nj-F_{2x}i-F_{2y}j+(78{,}34+7888{,}8\omega_1^2)i-(-17{,}4+1378{,}7\omega_1^2)j-(16{,}1\times10^{-3})j=37{,}43\omega_1^2i-1773{,}13\omega_1^2j$$

$$-F_{2x}+78{,}34+7888{,}8\omega_1^2=37{,}43\omega_1^2$$

$$F_{2x}=78{,}34+7851{,}37\omega_1^2$$

$$15-F_{2y}+17{,}4-1378{,}7\omega_1^2-16{,}1\times10^{-3}=-1773{,}13\omega_1^2$$

$$F_{2y}=32{,}4+394{,}43\omega_1^2$$

<u>LINK 1</u>

$$\sum\vec{M}=0$$

$$\vec{M}_1+(\vec{R}_{G_3o_1}\times\vec{W}_1)+(\vec{R}_{G_3o_1}\times(m_1\vec{a}_{G_1}))+\vec{R}_{o_2o_1}\times\vec{F}_2+(-I_1\alpha_1)=0$$

$$\vec{M}_1+((-29{,}1i+29{,}1j)\times(-34{,}31\times10^{-3}\,j))+((-29{,}1i+29{,}1j)\times(0{,}1\omega_1^2i-0{,}1\omega_1^2\,j))$$
$$+(-58{,}23i+58{,}23j)\times((78{,}34+7851{,}37\omega_1^2)i+(32{,}4+394{,}43\omega_1^2)j)=0$$

$$\vec{M}_1+(1{,}0\hat{k})+(2{,}9\omega_1^2-2{,}9\omega_1^2)\hat{k}+(-1886{,}7-22967{,}7\omega_1^2)\hat{k}-(4561{,}7+461747\,\omega_1^2)\hat{k}=0$$

$$\vec{M}_1=6448{,}4+484714{,}7\omega_1^2$$

$$\sum\vec{F}=0$$

$$-F_{1x}i+F_{1y}j-W_1j+F_{2x}i+F_{2y}j=0{,}1\omega_1^2i-0{,}1\omega_1^2j$$

$$-F_{1x}+F_{2x}=0{,}1\omega_1^2\ \Rightarrow\ F_{1x}=78{,}34+7851{,}27\omega_1^2$$

$$F_{1y}-34{,}31\times10^{-3}+F_{2y}=-0{,}1\omega_1^2\Rightarrow\ F_{1y}=-32{,}36-394{,}53\omega_1^2$$

6-CONCLUSION

This Project consists of calculation, drawing and manufacturing parts.Firstly, in calculation part the lengths of links were found.Then kinematic synthesis , dynamic synthesis and force analysis of the mechanisms were done .

In order to begin manufacturing part , we designed the mechanism and drew by using Solidworks.

Consequently, we started to manufacture the parts of our mechanism with respect to technical drawing.Then we assemble the parts and servomotors purchased. So we achieved to manufacture the prototype of knotting mechanism.

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